UDC 532.529+533.538

ON THE CONSTRUCTION OF MODELS OF POLARIZABLE DISPERSE AND MULTICOMPONENT MEDIA

PMM Vol.43, No.3, 1979, pp. 489-499

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(Received June 30, 1978)

A system of equations defining the motion of polarizable multiphase (disperse) media in an electromagnetic field is derived in approximation of electrohydrodynamics, using ideas of thermodynamics and hydrodynamics of polarizable media expounded in [1—4].

The system consists of equations of motion and energy for the mixture as a whole, of "diffusion" relationships that replace the equations of motion and energy for each phase, of equations of continuity and state, and of Maxwell equations. The medium is assumed to consist of k phases which can be taken as incompressible and of $N - k$ compressible phases, with each phase obeying its own polarization law and having its own temperature.

A formula is obtained for the force exerted by the electromagnetic field on the medium. The part of that force related to the process nonequilibrium contains derivatives of mean velocity, pressure differences, chemical potentials, and temperature of the various phases. The pressure of medium whose carrier phase, unlike the mixture of compressible phases [2], is incompressible, is not determined by thermodynamics of the medium but has to be derived from the solution of the problem.

Equations are derived for the variation of volume concentration of compressible phases, which relate variation of the true density of these phases to differences of pressures, chemical potentials, and to velocity gradients. The possibility of specifying internal energy as a function of dependent and independent parameters, and the equivalence of equations obtained in both cases is indicated. Equations for a polarizable incompressible homogeneous medium or a mixture of several incompressible phases are a particular result of the present investigation.

Equations of motions are derived for polarizable and magnetizable multicomponent media. The equations of diffusion in multicomponent media in a magnetic field when each component is magnetized according to its particular law are considered in detail.

The presence in disperse systems of phases that can be considered incompressible considerably complicates the description of such systems. This is due to that the introduction of parameters such as pressure, chemical potentials, entropy of phases, and of concepts such as density of energy and energy flux, etc. is determined by the system thermodynamics. The successive use of such method for defining the motion of a multiphase magnetizable medium in approximation of ferrohydrodynamics appeared in [2] in the case when all phases are compressible. The method used in [2] is inapplicable when part of the phases can be assumed incompressible. The presence of incompressible phases necessitates the introduction of new parameters unrelated to thermodynamics, and the redefinition of the previously used parameters of the system such as pressure, etc. Hence the formulas for forces acting on the medium, the work of these forces, diffusion relationships, and other formulas and equations that define the multicomponent media which contain incompressible phase, differ considerably from those derived in [2].

1. Derivation of equations for determining the motion of magnetizable disperse media. Let us consider the motion in an electric field of a disperse polarizable medium consisting of N phases: the carrier gas or liquid in which are dispersed $N-1$ kinds of charged phases (drops, bubbles, solid particles, etc.). Each of the phases may generally polarize according to its particular law.

We assume that the phases posess some conductivities, and that the medium contains two kinds of charged particles: free positive ions and negative ions or electrons. We assume that in addition to these free charged particles there are elementary charges stuck to dispersed particles and not moving relative to the latter (dispersed particles are charged). We further assume that in the considered medium k phases can be considered incompressible and the remaining $N-k$ phases compressible. We denote by numbers $1, 2, \ldots, k$ $(k+1, k+2, \ldots, N)$ the parameters related to incompressible (compressible) phases, and by numbers $N+1$, $N+2$ the parameters related to free ions charged positively and negatively, respectively.

We assume that phase α occupies in a unit volume the volume Γ_α $(\Gamma_1 + \Gamma_2 + \ldots + \Gamma_N = 1)$ and that its density is $\rho_\alpha{}^\circ$ (the true density of a phase). We introduce the temperature T_α, unit mass entropy s_α, mean velocity $\mathbf{v}_\alpha$, and the blurred density $\rho_\alpha = \Gamma_\alpha \rho_\alpha{}^\circ$ of each phase, and assume that ions and electrons occupy the whole volume, i.e. $\Gamma_{N+1} = \Gamma_{N+2} = 1$.

We denote by q_α the volume charge of phase α and of the free positively and negatively charged particles, and by $\mathbf{j}_\alpha = q_\alpha \mathbf{v}_\alpha$ the electric current density of the α-phase or of the free charges. When speaking of some parameter with subscript α or of equations containing parameters with the α-subscript, we understand, unless otherwise stated, that the respective parameter or equation are determined for α varying from 1 to $N+2$. The quantities $\rho_\alpha{}^\circ =$ const when $\alpha = 1, 2, \ldots, k$. The quantity $q_\eta = 0$; and subscript η denotes here and subsequently parameters related to the dispersing (carrier) phase.

We define the mean density ρ, the mixture velocity $\mathbf{u}$, the charge density q, and the total current $\mathbf{j}$ by formulas

$$\rho = \sum \rho_\alpha, \quad \rho \mathbf{u} = \sum \rho_\alpha \mathbf{v}_\alpha, \quad q = \sum q_\alpha, \quad \mathbf{j} = \sum \mathbf{j}_\alpha \tag{1.1}$$

Here and in what follows Σ and $\Sigma_{\alpha \neq \beta}$ denote, unless otherwise stated, summations carried out, respectively, from 1 to $N+2$, and from 1 to $N+2$ except for $\alpha = \beta$.

We shall consider the motion of the medium in approximation of electrohydrodynamics [3, 5, 6] in which the vectors of electric field and of electrical induction do not undergo transformation when passing from one inertial system to another. In the considered approximation Maxwell equations are of the form

$$\operatorname{rot} \mathbf{E} = 0, \quad \operatorname{aiv} \mathbf{D} = 4\pi q \tag{1.2}$$
$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}, \quad \operatorname{div} \mathbf{B} = 0$$

We assume that the internal energy U of the medium unit mass is a function of entropy of each phase and of free charged particles taken per unit mass of the mixture $c_\alpha s_\alpha$ $(\alpha = 1, 2, \ldots, N + 2)$, of the mixture density ρ, of the concentration of phases and free charged particles $c_\alpha = \rho_\alpha / \rho$ $(\alpha = 1, 2, \ldots, N + 2)$, volume concentration of phases Γ_α $(\alpha = 1, 2, \ldots, N)$, and of the absolute magnitude of the electrical unduction vector $\mathbf{D}$, i.e.

$$U = U(c_\alpha s_\alpha, \rho, c_\alpha, \Gamma_\alpha, D) \tag{1.3}$$

where the kinetic energy of the carrier phase associated with the pulsation of dispersed phases is disregarded, and the dependence of the system internal energy on the interface size of phases is not taken into consideration for the sake of brevity.

Variation of internal energy is defined by

$$dU = \sum T_\alpha dc_\alpha s_\alpha + \frac{p^c}{\rho^2} d\rho + \tag{1.4}$$
$$\sum \xi_\alpha dc_\alpha - \sum \frac{p_\alpha}{\rho} d\Gamma_\alpha + \frac{E_k dD_k}{4\pi\rho}$$
$$T_\alpha = \frac{\partial U}{\partial c_\alpha s_\alpha}, \quad \frac{p^c}{\rho^2} = \frac{\partial U}{\partial \rho}, \quad \xi_\alpha = \frac{\partial U}{\partial c_\alpha}, \quad \frac{p_\alpha}{\rho} = -\frac{\partial U}{\partial \Gamma_\alpha}$$
$$\frac{E_k}{4\pi\rho} = \frac{\partial U}{\partial D_k}$$

where and in what follows summation is carried out with respect to Latin subscripts from one to three. It is assumed that everywhere partial derivatives of any function f of several specified parameters are determined one by one, while the remaining ones are assumed constant, for instance,

$$\frac{\partial f(c_\alpha s_\alpha, \rho, c_\alpha, \Gamma_\alpha, D)}{\partial c_\gamma s_\gamma} = \left(\frac{\partial f}{\partial c_\gamma s_\gamma}\right)_{c_\alpha s_\alpha (\alpha \neq \gamma), \rho, c_\alpha, \Gamma_\alpha, D}$$

The last of equalities (1.4) implies that vectors of electric field and electrical induction are parallel: $\mathbf{D} = \varepsilon \mathbf{E}$, $\varepsilon = (8\pi\rho \partial U / \partial D^2)^{-1}$. The medium permittivity is generally a function of parameters $c_\alpha s_\alpha$, ρ, c_α, Γ_α, and D that determine the internal energy.

The equations of continuity for the density of phases and components, mixture density ρ, concentrations c_α, and for each phase charge q_α are of the form

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{u} = 0, \quad \rho \frac{dc_\alpha}{dt} = \varkappa_\alpha^\rho - \operatorname{div} \mathbf{J}_\alpha, \quad \mathbf{J}_\alpha = \rho_\alpha (\mathbf{v}_\alpha - \mathbf{u}) \tag{1.5}$$
$$\frac{\partial q_\alpha}{\partial t} + \operatorname{div} \mathbf{j}_\alpha = \varkappa_\alpha^q \quad (\alpha = 1, 2, \ldots, N, \ \alpha \neq \eta), \quad q_\eta = 0$$
$$q_{N+1\,(N+2)} = \frac{z_{N+1\,(N+2)} e_{N+1\,(N+2)} \rho_{N+1\,(N+2)}}{m_{N+1\,(N+2)}}, \quad \sum \varkappa_\alpha^\rho = 0$$

where $z_{N+1(N+2)}$ and $m_{N+1(N+2)}$ are the multiplicity and mass of positive (negative) free charges $e_{N+1} = e$ and $e_{N+2} = -e$, respectively, e is the proton charge, and $\varkappa_\alpha^p$ and $\varkappa_\alpha^q$ are the rates of creation of the mass and charge of the α-phase or component.

From the first and second of Eqs. (1.5) we obtain for the Γ_α incompressible phases the equation

$$d\Gamma_\alpha / dt + \Gamma_\alpha \operatorname{div} \mathbf{u} = (\varkappa_\alpha^\rho - \operatorname{div} \mathbf{J}_\alpha) / \rho_\alpha^\circ, \quad \alpha = 1, \ldots, k \tag{1.6}$$

Neglecting the field momentum and (for simplicity) the force of gravity, we can write the equation of momentum for the medium and field in the form

$$\partial \rho u_i / dt = -\partial (\rho u_i u_k - p_{ik}) / \partial x_k \tag{1.7}$$

We introduce the mixture pressure p and express the stress tensor p_{ik} in the form

$$p_{ik} = -p\delta_{ik} + \tau_{ik} \tag{1.8}$$

and pressure p in the form

$$p = p^c + p^{inc} \tag{1.9}$$

where p^{inc} is the part of mixture pressure that is unrelated to the thermodynamic parameters of compressible phases. The reason for introducing the term p^{inc} is to allow for the pressure of incompressible phases, not accounted for in thermodynamic functions. When all phases are compressible there is no need to introduce pressure p^{inc} in the medium governing parameters. Thus case was considered in [2].

Using the Gibbs identity (1.4) and Eqs. (1.5) and (1.7)–(1.9) we obtain

$$\begin{aligned} \frac{\partial}{\partial t}\left(\rho \frac{u^2}{2} + \rho U\right) = &-\frac{\partial}{\partial x_k}\left[\rho u_k \left(\frac{u^2}{2} + U + \frac{p}{\rho}\right) + \right. \\ &\left. \frac{c}{4\pi}[\mathbf{EH}]_k + q_k - u_i \Pi_{ik}\right] + \sum \rho T_\alpha \frac{dc_\alpha s_\alpha}{dt} + p^{inc} \operatorname{div} \mathbf{u} + \operatorname{div} \mathbf{q} + \\ &\sum \xi_\alpha (\varkappa_\alpha^\rho - \operatorname{div} \mathbf{J}_\alpha) - (\mathbf{j} - q\mathbf{u})\mathbf{E} - \sum_{\alpha=1}^{N} p_\alpha \frac{d\Gamma_\alpha}{dt} - \Pi_{ik} \nabla_k u_i + \\ &u_i \frac{\partial}{\partial x_k}\left(\tau_{ik} - \frac{E_i D_k}{4\pi} + \frac{\mathbf{ED}}{4\pi}\delta_{ik} - \Pi_{ik}\right) \end{aligned} \tag{1.10}$$

The term $\partial (q_k - u_i \Pi_{ik}) / \partial x_k$ has been added and subtracted from the right-hand side of formula (1.10). The physical meaning and expression for vector $\mathbf{q}$ and tensor Π_{ik} appear below.

As in [2], we made three assumptions that define the model.

1°. The total energy of the medium and field, and the flow of energy to the medium and field are, respectively,

$$\rho \frac{u^2}{2} + \rho U, \quad \rho \mathbf{u}\left(\frac{u^2}{2} + U + \frac{p}{\rho}\right) + \frac{c}{4\pi}[\mathbf{EH}] + \mathbf{q} - u_i \Pi_{ik} \mathbf{\ni}_k \tag{1.11}$$

where $\mathbf{\ni}_k$ are unit vectors of the basis. The equation of energy of the medium and field is of the form

$$\begin{aligned} \frac{\partial}{\partial t}\left(\rho \frac{u^2}{2} + \rho U\right) = &-\operatorname{div}\left[\rho \mathbf{u}\left(\frac{u^2}{2} + U + \frac{p}{\rho}\right) + \right. \\ &\left. \frac{c}{4\pi}[\mathbf{EH}] + \mathbf{q} - u_i \Pi_{ik} \mathbf{\ni}_k\right] \end{aligned} \tag{1.12}$$

2° . The equation of variation of the medium entropy is of the form

$$\sum \rho T_\alpha \frac{dc_\alpha s_\alpha}{dt} - p^{inc}\,\mathrm{div}\,\mathbf{u} + \mathrm{div}\,\mathbf{q} + \sum \xi_\alpha (\varkappa_\alpha{}^\rho - \mathrm{div}\,\mathbf{J}_\alpha) - \tag{1.13}$$

$$(\mathbf{j} - q\mathbf{u})\,\mathbf{E} - \sum_{\alpha=1}^{N} p_\alpha \frac{d\Gamma_\alpha}{dt} - \Pi_{ik}\nabla_k u_i = 0$$

It follows from Eqs. (1.10), (1.12), and (1.13) that

$$\frac{\partial}{\partial x_k}\tau_{ik} = \frac{\partial}{\partial x_k}\left(\frac{E_i D_k}{4\pi} - \frac{\mathbf{ED}}{4\pi}\delta_{ik} + \Pi_{ik}\right) \tag{1.14}$$

3°. The dissipation function σ is of the form

$$\sigma = \sum_{\alpha\neq\eta} \frac{\rho_\alpha}{T_\eta}\frac{d_\alpha s_\alpha}{dt}(T_\eta - T_\alpha) + \sum_{\alpha\neq\eta}\frac{\varkappa_\alpha{}^\rho l_\alpha}{T_\eta} + \tag{1.15}$$

$$\sum_{\substack{\alpha=k+1\\ \alpha\neq\eta}}^{N} \frac{\rho_\alpha}{T_\eta}\frac{d_\alpha(\Gamma_\alpha/\rho_\alpha)}{dt}(p_\alpha{}^* - p_\eta{}^*) + \sum_{\alpha\neq\eta}\mathbf{J}_\alpha\zeta_\alpha - \mathbf{q}\frac{\nabla T_\eta}{T_\eta{}^2} +$$

$$\left\{\Pi_{ik} - \sum_{\substack{\alpha=1\\ \alpha\neq\eta}}^{N}\Gamma_\alpha(p_\alpha{}^* - p_\eta{}^*)\,\delta_{ik}\right\}\frac{\nabla_k u_i}{T_\eta}$$

$$\xi_\alpha{}^* = \xi_\alpha, \quad p_\alpha{}^* = p_\alpha \ (\alpha = k+1, \ldots, N+2)$$

$$\xi_\alpha{}^* = \xi_\alpha + p^{inc}\left(\rho_\alpha{}^\circ \sum_{\gamma=1}^{k}\Gamma_\gamma\right)^{-1}$$

$$p_\alpha{}^* = p_\alpha + p^{inc}\left(\sum_{\gamma=1}^{k}\Gamma_\gamma\right)^{-1} \quad (\alpha = 1, 2, \ldots, k)$$

$$l_\alpha = \xi_\eta{}^* - \xi_\alpha{}^* + s_\alpha(T_\eta - T_\alpha) + (p_\alpha{}^* - p_\eta{}^*)\frac{\Gamma_\alpha}{\rho_\alpha}$$

$$(\alpha \neq N+1, \quad N+2, \ \eta)$$

$$l_\alpha = \xi_\eta{}^* - \xi_\alpha{}^* + s_\alpha(T_\eta - T_\alpha)\,(\alpha = N+1, \ N+2)$$

$$\zeta_\alpha = \nabla\frac{\xi_\eta{}^* - \xi_\alpha{}^*}{T_\eta} + \frac{q_\alpha\mathbf{E}}{\rho_\alpha T_\eta} + s_\alpha\nabla\frac{T_\eta - T_\alpha}{T_\eta} + \frac{\Gamma_\alpha}{\rho_\alpha}\times$$

$$\nabla\frac{p_\alpha{}^* - p_\eta{}^*}{T_\eta} \quad (\alpha\neq\eta, \ N+1, \ N+2)$$

$$\zeta_\alpha = \nabla\frac{\xi_\eta{}^* - \xi_\alpha{}^*}{T_\eta} + \frac{q_\alpha\mathbf{E}}{\rho_\alpha T_\eta} + s_\alpha\nabla\frac{T_\eta - T_\alpha}{T_\eta}(\alpha = N+1, \ N+2)$$

Formula (1.15) for the dissipation function substantially differs from the corresponding formula (1.20) in [2]. This is due, first, to that a polarizable medium is considered here, not a magnetizable one as in [2], and, second, to the introduction of pressure p^{inc} in the determining parameters of the system.

Defining the mixture entropy S by formula $S = \Sigma c_\alpha s_\alpha$ and taking into account the expression for σ in (1.15) and the definition $\mathbf{j} - q\mathbf{u} = \Sigma q_\alpha \mathbf{J}_\alpha / \rho_\alpha$ we obtain from Eqs. (1.13) and (1.15) for the mixture entropy variation an equation of the form

$$\frac{\partial}{\partial t}\rho S = -\frac{\partial}{\partial x_k}\left\{\rho u_k S + \frac{q_k}{T_\eta} + \frac{1}{T_\eta}\sum_{\alpha\neq\eta} J_{\alpha k} l_\alpha\right\} + \sigma \tag{1.16}$$

Applying the Curie theorem and the Onsager rule we obtain

$$\mathbf{J}_\alpha = \sum_{\nu\neq\eta} L_{\alpha,\,\nu}\zeta_\nu + L_{\alpha,\,N+3}\nabla T_\eta \tag{1.17}$$

$$-\frac{\mathbf{q}}{T_\eta^2} = \sum_{\nu\neq\eta} L_{N+3,\,\nu}\zeta_\nu + L_{N+3,\,N+3}\nabla T_\eta \tag{1.18}$$

$$\frac{1}{T_\eta}\,\rho_\alpha\frac{d_\alpha s_\alpha}{dt} = \sum_{\nu\neq\eta}\varphi_{\alpha,\,\nu}(T_\eta - T_\nu) + \sum_{\nu\neq\eta}\varphi_{\alpha,\,N+2+\nu} l_\nu + \sum_{\substack{\nu=k+1\\ \nu\neq\eta}}^{N}\varphi_{\alpha,\,2N+4+\nu}(p_\nu{}^* - p_\eta{}^*) + \varphi_{ij}{}^\alpha\nabla_i u_j, \quad \alpha\neq\eta \tag{1.19}$$

$$\frac{\varkappa_\alpha{}^\rho}{T_\eta} = \sum_{\nu\neq\eta}\varphi_{N+2+\alpha,\,\nu}\ (T_\eta - T_\nu) + \sum_{\nu\neq\eta}\varphi_{N+2+\alpha,\,N+2+\nu} l_\nu + \sum_{\substack{\nu=k+1\\ \nu\neq\eta}}^{N}\varphi_{N+2+\alpha,\,2N+4+\nu}(p_\nu{}^* - p_\eta{}^*) + \varphi_{ij}^{N+2+\alpha}\nabla_i u_j, \quad \alpha\neq\eta \tag{1.20}$$

$$\frac{\rho_\alpha}{T_\eta}\frac{d_\alpha(\Gamma_\alpha/\rho_\alpha)}{dt} = \sum_{\nu\neq\eta}\varphi_{2N+4+\alpha,\,\nu}(T_\eta - T_\nu) + \sum_{\nu\neq\eta}\varphi_{2N+4+\alpha,\,N+2+\nu} l_\nu + \sum_{\substack{\nu=k+1\\ \nu\neq\eta}}^{N}\varphi_{2N+4+\alpha,\,2N+4+\nu}(p_\nu{}^* - p_\eta{}^*) + \varphi_{ij}^{2N+4+\alpha}\nabla_i u_j, \quad \alpha = k+1,\ k+2,\ldots,N,\ \alpha\neq\eta \tag{1.21}$$

$$T_\eta^{-1}\left\{\Pi_{ik} - \sum_{\substack{\alpha=1\\ \alpha\neq\eta}}^{N}\Gamma_\alpha(p_\alpha{}^* - p_\eta{}^*)\,\delta_{ik}\right\} = \sum_{\nu\neq\eta}\Psi_{ik}^{\nu}(T_\eta - T_\nu) + \sum_{\nu\neq\eta}\Psi_{ik}^{N+2+\nu} l_\nu + \sum_{\substack{\nu=k+1\\ \nu\neq\eta}}^{N}\Psi_{ik}^{2N+4+\nu}(p_\nu{}^* - p_\eta{}^*) + L_{ikjl}\nabla_j u_l \tag{1.22}$$

Equations (1.17) and (1.20) are valid for $\alpha\neq\eta$, but, since $\Sigma\varkappa_\alpha{}^\rho = 0$ and $\Sigma\mathbf{J}_\alpha = 0$, it is possible to write the expression for $\mathbf{J}_\eta$ and $\varkappa_\eta{}^\rho$ in a form similar to that in [2], and assume that these equations are also valid for $\alpha = \eta$.

When the electric field [instensity] is fairly low and the medium can be considered isotropic, the coefficients in Eqs. (1.17)–(1.22) are assumed independent of the electric field and to satisfy the usual Onsager reciprocity relationships for an isotropic medium

$$
\begin{gathered}
L_{\alpha,\mu} = L_{\mu,\alpha}, \quad L_{\alpha,N+3} = L_{N+3,\alpha} \\
(\alpha, \mu = 1, 2, \ldots, N+2; \quad \alpha, \mu \neq \eta); \quad \varphi_{\alpha,\gamma} = \varphi_{\gamma,\alpha} \\
\varphi_{ij}{}^{\alpha} = -\psi_{ij}{}^{\alpha} \, (\alpha, \gamma = 1, 2, \ldots, 3N+4; \ \alpha, \gamma \neq \eta), N+2+\eta, \\
2N+5 \ldots, 2N+4+k, 2N+4+\eta); \, L_{ijkl} = L_{klij}
\end{gathered}
\tag{1.23}
$$

Equations (1.17) are used for the determination of vectors of diffusion, and (1.19) for determining temperatures (or entropies) or phases and components, replacing the unwieldy equations of energy for each phase and component. The kinetic equations (1.17) can be written in the form of Ohm's laws for the motion of phase and component charges, as was done in [2].

For $\alpha = k+1, \ldots, N, \alpha \neq \eta$ Eqs. (1.21) make it possible to determine the rate of volume concentration variation of compressible phases, without having to resort to equations of the Rayleigh type for bubble pulsations, which on the assumptions made here does not follow from the applied formalizm of thermodynamics of incompressible processes.

Denoting by U_0 the internal energy of the medium in the case of absence of a field, from the last of formulas (1.4) we obtain

$$
U = U_0 + \frac{1}{4\pi\rho} \int_0^D \mathbf{E} d\mathbf{D} \tag{1.24}
$$

Taking into account formulas (1.8), (1.14), (1.24), and (1.4) for the stress tensor and force, the chemical potential ξ_α, and pressure p_α we obtain expressions of the form

$$
\begin{gathered}
p_{ik} = -\left(p^c + p^{inc} + \frac{\mathbf{ED}}{4\pi}\right)\delta_{ik} + \frac{E_i D_k}{4\pi} + \Pi_{ik}, \qquad f_i = \frac{\partial p_{ik}}{\partial x_k} \\
p^c = \rho^2 \frac{\partial U}{\partial \rho} = p_0{}^c - \frac{1}{4\pi} \int_0^D \left(1 + \frac{\rho}{\varepsilon} \frac{\partial \varepsilon}{\partial \rho}\right) \frac{\mathbf{D} d\mathbf{D}}{\varepsilon}, \qquad p_0{}^c = \rho^2 \frac{\partial U_0}{\partial \rho} \\
p_\alpha = p_{0\alpha} + \frac{1}{4\pi} \int_0^D \frac{\partial \varepsilon}{\partial \Gamma_\alpha} \frac{\mathbf{D} d\mathbf{D}}{\varepsilon^2}, \qquad p_{0\alpha} = -\rho \frac{\partial U_0}{\partial \Gamma_\alpha} \\
\xi_\alpha = \xi_{0\alpha} - \frac{1}{4\pi\rho} \int_0^D \frac{\partial \varepsilon}{\partial c_\alpha} \frac{\mathbf{D} d\mathbf{D}}{\varepsilon^2}, \qquad \xi_{0\alpha} = \frac{\partial U_0}{\partial c_\alpha}
\end{gathered}
\tag{1.25}
$$

When the internal energy U is specified as a function of true densities $\rho_\alpha{}^\circ$ and volume concentrations Γ_α (not $\rho, c_\alpha, \Gamma_\alpha$) and of parameters c_α s_α, and D, the formulas for p^c, p_α and ξ_α in (1.25) assume the form

$$
\begin{gathered}
p^c = p_0{}^c - \frac{1}{4\pi} \int_0^D \left[1 + \frac{1}{\varepsilon} \sum_{\alpha=k+1}^{N+2} \rho_\alpha{}^\circ \frac{\partial \varepsilon}{\partial \rho_\alpha{}^\circ}\right] \frac{\mathbf{D} d\mathbf{D}}{\varepsilon} \\
p_\alpha = p_{0\alpha} + \frac{1}{4\pi} \int_0^D \left(\frac{\partial \varepsilon}{\partial \Gamma_\alpha} - \frac{\rho_\alpha{}^\circ}{\Gamma_\alpha} \frac{\partial \varepsilon}{\partial \rho_\alpha{}^\circ}\right) \frac{\mathbf{D} d\mathbf{D}}{\varepsilon^2}
\end{gathered}
\tag{1.26}
$$

$$\xi_\alpha = \xi_{0\alpha} - \frac{1}{4\pi}\int_0^D \frac{1}{\Gamma_\alpha}\frac{\partial\varepsilon}{\partial\rho_\alpha{}^\circ}\frac{\mathbf{D}d\mathbf{D}}{\varepsilon^2}$$

In the case, when in a two-phase uncharged medium $(q_\alpha = 0,\ \alpha = 1,\ 2)$ it is possible to disregard cross effects and the phase temperatures are the same, the equation for diffusion is of the form

$$\mathbf{J}_\alpha = L_{\alpha,\alpha}\left[\nabla\frac{\xi_\eta{}^* - \xi_\alpha{}^*}{T_\eta} + \frac{1}{\rho_\alpha{}^\circ}\nabla\frac{p_\alpha{}^* - p_\eta{}^*}{T_\eta}\right], \alpha = 1, 2, \qquad \alpha \neq \eta \quad (1.27)$$

If both phases are compressible $(k = 0)$, formula (1.27) with allowance for expressions for $\xi_\alpha{}^*$ and $p_\alpha{}^*$ in (1.15) and (1.25), respectively, (with $\eta = 2$ for definites) assumes the form

$$L_{1,1}^{-1}\mathbf{J}_1 = \nabla\frac{\xi_{02} - \xi_{01}}{T_2} + \nabla\int_0^D\left(\frac{\partial\varepsilon}{\partial c_1} - \frac{\partial\varepsilon}{\partial c_2}\right)\frac{\mathbf{D}d\mathbf{D}}{4\pi\rho\varepsilon^2 T_2} + \qquad (1.28)$$

$$\frac{1}{\rho_1{}^\circ}\nabla\frac{p_{01} - p_{02}}{T_2} + \frac{1}{\rho_2{}^\circ}\nabla\frac{1}{T_2}\int_0^D\left(\frac{\partial\varepsilon}{\partial\Gamma_1} - \frac{\partial\varepsilon}{\partial\Gamma_2}\right)\frac{\mathbf{D}d\mathbf{D}}{4\pi\varepsilon^2}$$

At small concentrations of the dispersed phase the mixture permittivity ε is defined in terms of permittivities of phases by formula

$$\varepsilon = \varepsilon_2 + \Gamma_1\frac{3(\varepsilon_1 - \varepsilon_2)\,\varepsilon_2}{\varepsilon_1 + 2\varepsilon_2} \qquad (1.29)$$

where ε_2 and ε_1 are the permittivities of the carrier and dispersed phases, respectively.

Let us consider the diffusion of gas bubbles in a compressible liquid whose permittivities are defined by formulas

$$(\varepsilon_2 - 1)\,/\,(\varepsilon_2 + 2) = \alpha_2\rho_2{}^\circ, \quad \varepsilon_1 = 1 + \alpha_1\rho_1{}^\circ \qquad (1.30)$$

Equation (1.28) with allowance for the equalities (1.29) and (1.30) assumes the form

$$L_{1,1}^{-1}\mathbf{J}_1 = \nabla\frac{\xi_{02} - \xi_{01}}{T_2} + \frac{1}{\rho_1{}^\circ}\nabla\frac{p_{01} - p_{02}}{T_2} + \qquad (1.31)$$

$$\left\{\frac{9\alpha_1\varepsilon_2{}^2}{(\varepsilon_1 + 2\varepsilon_2)^2} - \frac{\alpha_2(\varepsilon_2 + 2)^2}{3\Gamma_1}\left[1 + \frac{3\Gamma_1(\varepsilon_1{}^2 - 2\varepsilon_2\varepsilon_1 - 2\varepsilon_2{}^2)}{(\varepsilon_1 + 2\varepsilon_2)^2}\right]\right\} \times$$

$$\frac{E^2}{8\pi T_2}\nabla\frac{1}{\rho_1{}^\circ} + \frac{1}{\rho_1{}^\circ}\nabla\frac{3(\varepsilon_1 - \varepsilon_2)\,\varepsilon_2}{\varepsilon_1 + 2\varepsilon_2}\frac{E^2}{8\pi T_2}$$

We assume that the internal energy of the disperse medium and of the field per unit of medium mass U is specified, i.e. that functions U_0 and $\varepsilon\,(c_\alpha s_\alpha,\ \rho,\ c_\alpha,\ \Gamma_\alpha,\ D)$ are known. It is then possible to formulate a closed system of equations for defining the motion of the disperse mixture in the electric field when each phase and component are polarized according to their particular laws. In the approximation of electrohydrodynamics the system of equations consists of continuity equations (1.5), equations of state (1.4), the equation of motion of the mixture (1.7) with allowance for formula (1.8) for p_{ij}, Eqs. (1.9), (1.14), and (1.22), of the equation of energy for the mixture (1.12), kinetic equations (1.17) – (1.21), and Maxwell equations (1.2).

Note that pressure p^{inc} for $\eta > k$, i.e. when the carrier phase is compressible, does not appear in the derived system of equations, and there is no need to formulate for it any additional relations. However, when $1 \leqslant \eta \leqslant k$ pressure p^{inc} is represented in the system of equations, its determination requires in addition to all equations the equality $\Gamma_1 + \Gamma_2 + \ldots + \Gamma_N = 1$.

To determine the adsorption charge of each phase q_α $(\alpha = 1, \ldots, N,\ \alpha \neq \eta)$ it is necessary to make additional assumptions about the rate of charge variation $\varkappa_\alpha{}^q$.

In constructing the model it was assumed above that the internal energy U is a function of variables among which c_α, Γ_α $(\alpha = 1, \ldots, N)$ and ρ are dependent. [1]. This approach makes possible to write all formulas in a convenient symmetric form. It is possible to write an equation in which the internal energy $\bar{U}$ is a function of only independent variables, and to show that the equations obtained with the use of function $\bar{U}$ are exactly the same as those in whose derivation the internal energy U was used [4]. To pass from function U to function U it is necessary to use formulas that link the parameters on which depends the internal energy, so as to eliminate a part of parameters from the expression for the internal energy U [4].

In actual application of the derived above equations one has to bear in mind that the form of equations substantially depends on the selection of parameters that define the internal energy. When the internal energy is defined in terms of only a part of parameters, or of some combination of these (e.g., s_α, $\rho_\alpha{}^\circ$, Γ_α, and D / ρ or s_α, ρ, c_α, Γ_α, and D / ρ, etc.), the use of equations derived here requires that the internal energy be specified as a function of parameters $c_\alpha s_\alpha$, ρ, c_α, Γ_α, and D used here, and apply this transformed function in all formulas.

2. Equations of motion of polarizable multicomponent media. Let us consider in the approximation of electrohydrodynamics [3, 5, 6] the motion of a mixture consisting of N components filling one and the same volume, each of which polarizes in the electric field $\mathbf{E}$ according to its particular law.

We denote by ρ_α, $\mathbf{v}_\alpha$, T_α, s_α, and q_α, respectively, the density velocity vector, temperature, entropy per unit mass of each component, and the volume charge density of the α component; the electric current density of the α component will be denoted by $\mathbf{j}_\alpha$. We determine the mixture mean density ρ, its mean velocity $\mathbf{u}$, total charge, and current by formulas (1.1).

We assume that the internal energy U per unit mass of medium and field is a function of $c_\alpha s_\alpha$, ρ, c_α and D. On the assumptions made above in Sect. 1 about the density of energy and energy flux, and the dissipation function, it is possible to derive equations for defining the motion of the considered medium.

The equations of continuity are the same as the corresponding equations (1.5), and the equation of motion is analogous to Eq. (1.7) in which the stress tensor p_{ik} is of the form of the first of formulas (1.25) in which $p = \rho^2 (\partial U / \partial \rho)_{c_\alpha s_\alpha, c_\alpha, D}$ is substituted for $p^c + p^{inc}$.

The equation of energy becomes the same as Eq. (1.12) if in the latter we set $p = \rho^2 (\partial U / \partial \rho)$, and the form of the dissipation functions becomes the same as the form of the dissipation function (1.15), after elimination from the latter of terms proportional to $p_\alpha{}^* - p_\eta{}^*$ and the substitution of $(\partial U / \partial c_\alpha)_{c_\gamma s_\gamma, \rho, c_\gamma (\gamma \neq \alpha), D}$ for $\xi_\alpha{}^*$.

The kinetic equations which correspond to this dissipation function are generally different from kinetic equations that define a multiphase medium. The final system of equations defining the motion of multicomponent medium is of the form

$$\frac{\partial \rho_\alpha}{\partial t} + \operatorname{div} \rho_\alpha \mathbf{u} = \varkappa_\alpha - \operatorname{div} \mathbf{J}_\alpha, \quad \mathbf{J}_\alpha = \rho_\alpha (\mathbf{v}_\alpha - \mathbf{u}), \quad \sum_{\alpha=1}^{N} \varkappa_\alpha = 0$$

$$\frac{\partial}{\partial t} \rho u_i = - \frac{\partial}{\partial x_k} (\rho u_i u_k - p_{ik})$$

$$p_{ik} = - p\delta_{ik} + \Pi_{ik} + \frac{E_i D_k}{4\pi} - \frac{\mathbf{ED}}{4\pi} \delta_{ik}, \quad p = \rho^2 \left(\frac{\partial U}{\partial \rho}\right)_{c_\alpha s_\alpha, c_\alpha, D}$$

$$\frac{\partial}{\partial t} \left(\rho \frac{u^2}{2} + \rho U\right) = - \frac{\partial}{\partial x_k} \left\{\rho u_k \left(\frac{u^2}{2} + U + \frac{p}{\rho}\right) + \frac{c}{4\pi} [\mathbf{EH}]_k + \right.$$

$$\left. q_k - \Pi_{ik} u_i \right\}$$

$$U = U (c_\alpha s_\alpha, \rho, c_\alpha, D), \quad \xi_\alpha = (\partial U / \partial c_\alpha)_{c_\alpha s_\alpha, \rho, c_\gamma (\gamma \neq \alpha), D}$$

$$\frac{\rho_\alpha}{T_\beta} \frac{d_\alpha s_\alpha}{dt} = \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} \varphi_{\alpha, \nu} (T_\beta - T_\nu) + \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} \varphi_{\alpha, N+\nu}\, l_\nu + \varphi_{ik}^{\alpha} \nabla_k u_i, \quad \alpha \neq \beta$$

$$\frac{\varkappa_\alpha}{T_\beta} = \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} \varphi_{N+\alpha, \nu} (T_\beta - T_\nu) + \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} \varphi_{N+\alpha, N+\nu} l_\nu + \varphi_{ik}^{N+\alpha} \nabla_k u_i, \quad \alpha \neq \beta$$

$$\mathbf{J}_\alpha = \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} L_{\alpha, \nu} \xi_\nu + L_{\alpha, N+1} \nabla T_\beta$$

$$- T_\beta^{-2} \mathbf{q} = \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} L_{N+1, \nu}\, \xi_\nu + L_{N+1, N+1} \nabla T_\beta$$

$$T_\beta^{-1} \Pi_{ik} = L_{ikjl} \nabla_j u_l + \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} \Psi_{ik}^{\nu} (T_\beta - T_\nu) + \sum_{\substack{\nu=1 \\ \nu \neq \beta}}^{N} \Psi_{ik}^{N+\nu} l_\nu$$

$$l_\nu = \xi_\beta - \xi_\nu + s_\nu (T_\beta - T_\nu), \quad \nu \neq \beta; \; T_\alpha = (\partial U / \partial c_\alpha s_\alpha)_{c_\gamma s_\gamma (\gamma \neq \alpha), \rho, c_\gamma, D}$$

$$\zeta_\nu = \nabla \frac{\xi_\beta - \xi_\nu}{T_\beta} + \left(\frac{q_\nu}{\rho_\nu} - \frac{q_\beta}{\rho_\beta}\right) \frac{\mathbf{E}}{T_\beta} + s_\nu \nabla \frac{T_\beta - T_\nu}{T_\beta}, \quad \nu \neq \beta$$

(the corresponding system of equations for $\Pi_{ik} = 0$ can be found in [3]).

The coefficients in the last five of the above kinetic equations $\varphi_{\alpha,\nu}$, $\varphi_{ij}{}^{\alpha}$, $\Psi_{ij}{}^{\nu}$, $L_{\mu,\nu}$, and L_{ijkl} satisfy the usual Onsager reciprocity relationships.

It should be stressed that in the case of a polarizable multicomponent medium the stress tensor p_{ik} depends on polarization of the medium not only in terms of pressure p and tensor $E_i D_k / (4\pi) - \mathbf{ED}\delta_{ik} / (4\pi)$ but, also, in terms of the chemical potential difference contained in tensor Π_{ik}.

3. Equations defining the motion of magnetizable multiphase and multicomponent media. Derivation of these equations is similar to that in Sects. 1 and 2. The equations of conservation of mass, momentum, energy, and the equations of state and the kinematic equations for a magnetizable multiphase and multicomponent medium in the approximation of ferrohydrodynamics are the same as the equations for a polarizable multiphase and multicomponent medium in approximation of electrohydrodynamics after the substitution in the latter of $\mathbf{H}$ for $\mathbf{E}$, $\mathbf{B}$ for $\mathbf{D}$, μ for ε, $\mathbf{j}$ for $\mathbf{j}'$, and $\mathbf{E}' = \mathbf{E} + c^{-1}[\mathbf{uB}]$ for $\mathbf{E}$ (note that it is not necessary to carry out the substiution in the expression $c\,[\mathbf{EH}]\,/\,(4\pi)$ for Pointing's vector, the substitution $\mathbf{E}'$ for $\mathbf{E}$ is only made in the expression for ξ_α (1.15). In the case of magnetizable media the coefficients in kinetic equations are related by corresponding Onsager reciprocity relationships in a magnetic field [2]. Maxwell equations in approximation of ferrohydrodynamics appear in [3].

Let us consider diffusion in two-component magnetizable medium. In many instances it is possible to define the magnetic susceptibility of a two-component mixture by formula

$$\chi = \chi_1 + \chi_2 \qquad (3.1)$$

where χ_1 and χ_2 are the magnetic susceptibilities of the first and second component, respectively.

Formula (3.1) is obtained on the assumption of additivity of mixture magnetization

$$M = M_1 + M_2, \quad M_\alpha = \chi_\alpha H, \quad \alpha = 1, 2 \qquad (3.2)$$

Let us compare the formulas obtained here for diffusion in a multicomponent mixture with those in [7]. As in [7], we assume that the formulas

$$\chi_\alpha = c_\alpha \chi_{0\alpha}, \quad \chi_{0\alpha} = \text{const}, \quad \alpha = 1, 2 \qquad (3.3)$$

where $\chi_{0\alpha}$ is the true magnetic susceptibility of component α, hold for multicomponent mixtures. This formula is a corollary of Langevin's formula for magnetization intensity M_α of phase α, when the density of the multicomponent mixture ρ is constant, for instance, when the mixture mean velocity is zero and at the initial instant of time the mixture is homogeneous.

The formula defining the diffusion in a mixture of two magnetizable uncharged liquids at the same constant temperature is, with allowance for formulas (3.1) and (3.3), of the form

$$J_2 = -J_1 = L_{1,1}\left[\nabla\frac{\xi_{01} - \xi_{02}}{T_2} + \frac{\chi_{01} - \chi_{02}}{2\rho T_2}\nabla H^2\right], \quad \xi_{0\alpha} = \left(\frac{\partial U_0}{\partial c_\alpha}\right)_{c_\gamma s_\gamma,\, c_\gamma(\gamma \neq \alpha)} \qquad (3.4)$$

If the mixture temperature is constant in space, the first term in formula (3.4) is proportional to the concentration gradient and defines the diffusion generated by nonuniform concentration. The second term defines the diffusion of components in an inhomogeneous magnetic field due to the difference between magnetic susceptibility of mixture components. Formula (3.4) differs from the formula for diffusion in the magnetizable mixture of two liquids proposed in [7]. The formulas in [7] imply that component interdiffusion in an inhomogeneous magnetic field occurs, also, when the magnetic susceptibilities χ_{01} and χ_{02} of the two media are the same, while it follows from formulas (3.4) that in that case there is not interdiffusion between components in a magnetic field (diffusion induced by concentration gradient is evidently still present).

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Translated by J. J. D.